

On the existence of the optimal fine for environmental crime

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Abstract

Classical theory states that the optimal fine equals the damage caused by the crime divided by the probability of detection. But does such an optimal fine exist? We focus on emissions from production, and, even if we assume that the damage function is perfectly known, we still show that the optimal fine typically does not exist. Non-existence occurs as the environmental damage function is non-linear in overall emissions, meaning that there are interactions between emissions, the economy and the environment. We argue that these interactions cannot be accurately reflected in the fine imposed by the regulator. Previous literature on optimal fines does not recognize the non-existence of the applicable optimal fine, basically since this literature uses discrete models where the damage caused by the crime is assumed constant. Our result reduces the attractiveness of fines and might thus help explain enforcement agencies' inclination towards non-monetary enforcement instruments.

Keywords: Environmental crime; regulatory enforcement; penalty; optimal fine; judicial discretion

JEL classification: K42, Q28, L51, K32

1. Introduction

Environmental regulations, standards as well as taxes, need to be enforced, and even though several alternative enforcement instruments are available, the one most commonly discussed in the economics literature is the fine. In general the regulator sets a fine, and firms maximize profit given the expected fine for non-compliance. In this situation the regulator should set the expected fine to make social welfare as high as possible – typically balancing abatement costs and environmental damage costs. It is textbook knowledge that this implies that the optimal fine equals the damage caused by the crime, divided by the probability of apprehension (Polinsky and Shavell 2007).

To implement the optimal fine scheme in reality, the regulator would need to measure the damage function, a task recognized to be insurmountable (Baumol and Oates 1971). We focus on emissions from production, and, *even* if we assume that the shape and specification of the damage function is perfectly known ex-ante, we still show that an applicable and implementable optimal fine typically does not exist. Even if all contributing factors and their relative importance are known, this implies that it is very difficult to predict ex-ante the actual harm caused by one specific violation. Non-existence occurs as the environmental damage function is non-linear in overall emissions, meaning that there are non-trivial interactions between emissions, the economy and/or the environment. We argue that these complex interactions cannot be accurately reflected in the fine imposed by a regulator. Our result is particularly relevant since, as we will argue, most environmental pollution problems involve several polluters whose combined actions result in non-separable damages due to complex interactions between nature, society and human well-being. While we focus on environmental offenses, our model applies in all situations where the social damage from the offense is a non-linear function of the seriousness of the offense due to, for instance, threshold effects, or the impact of actions taken by other parties or victims to minimize the harm.

Fines are rarely the only enforcement instrument available to agencies and courts. A wide variety of instruments is used by regulators; such as prison sentences, regularization orders or suspension of environmental licenses and warnings. The extensive appliance of these second-best enforcement instruments have previously been explained by invoking notions like social costs of sanctioning (e.g. Kaplow 1990 and Shavell 1987), by restricting the size of the fine that can be imposed (e.g. Polinsky and Shavell 1991), or by acknowledging that the firms or the enforcement agency can commit errors (e.g. Nyborg and Telle 2004 and Rousseau 2009). We, however, point to a more fundamental motivation for providing law enforcers with the possibility of using non-monetary sanctions (and not only fines). Since fines cannot be optimally designed ex-ante for many environmental problems, the regulator cannot be certain about the effect of the regulation. The non-linearity of the damage function precludes the specification of the optimal fine function ex-ante and thus also the provision of the right deterrence incentives to potential violators. Violators cannot correctly anticipate the fine imposed (in court) before actually committing the violation, which is likely to lead to over- or underdeterrence. Thus non-optimal deterrence leads to too little or too much abatement by firms. A cautious regulator may therefore want to include some guarantees into the regulatory system to make sure that the environmental damage does not exceed certain thresholds. Non-monetary sanctions are typical examples of such risk reducing measures, since they go to the source of the problem and make sure that the polluting activity is effectively stopped by (temporarily) closing down the firm or incarcerating the violator.

We now turn to the previous literature on optimal fines. One strand of the literature, following Becker's seminal paper in 1968, points out that fines should be maximal, since raising fines is cheaper than increasing the likelihood of detection. However, in order to preserve marginal deterrence, Shavell (1992) and Mookherjee and Png (1994) argue that fines should be an increasing function of the severity of the violation(s), and thus maximal fines are not optimal.

Taking this into account, the literature provides unequivocal results (see, among others, Polinsky and Shavell 1979, 1992 and Garoupa 2001): the regulator should set the optimal fine equal to the harm caused corrected for the probability that the violator is sanctioned (while taking the costs of prosecution and detection into account). Our issue of non-existence of the optimal fine, however, does not occur in this previous work. The main reason for this non-occurrence is that, in general, the previous work applies discrete models where the harm caused by committing the crime is assumed constant (see e.g. review by Polinsky and Shavell 2007).¹ Other models looking at the influence of non-compliance on environmental policy tend to impose limitations in the model in order to ensure existence of a unique interior solution. These assumptions about the derivatives of the functions used in the models implicitly imply that environmental damage functions are constant or linear (see, for example, the assumption of a constant willingness-to-pay for an additional unit of emission reduction in Rousseau and Proost 2005).

The rest of the paper is laid out as follows. The next section starts by looking at the environmental damage function in more detail. In the third section we model the compliance and production decisions of the firms when they are faced with an emission standard. This allows us to clarify the conditions for existence of optimal fine functions. Moreover, we show that the optimal fine as a function of the prosecuted firm's abatement effort does not exist if damage functions are non-linear. In the fourth section, we show that the results hold for other environmental policy instruments as well. We discuss the conditions and implications of the non-existence result in the fifth section, where we also provide the case for alternative enforcement instruments. Finally, in section six we conclude.

2. Environmental damage functions

It has long been recognized that estimation of actual damage functions is an insurmountable task. These measurement problems mean that for most pollutants we simply do not have a reliable estimate of the damage function. Already Baumol and Oates (1971) commented on this lack of information and on its consequences for environmental policy.²

“Because we are unable to measure social welfare, and because we do not know the vector of inputs and outputs that characterize the optimum, we simply do not know whether a given change in the tax rate has moved us toward that optimum or has even

¹ Heyes' (2000) overview does not consider the issue of non-existence of the optimal fine. Cohen (2000) uses a more complicated fine function but by restricting the model to one firm the non-existence issue does not show up. In the literature on environmental tax schemes, consequences of non-linear damage functions (in overall emissions from several polluters) have received limited attention (Kaplow and Shavell 2002, note 13).

² Also in the legal profession, the difficulties in estimating environmental damage for liability and compensation issues have been widely recognized. Larsson (1999), for example, specifically mentions that the estimation of damage requires accurate scientific data on a) the levels and distributions of the pollutant including their proportions derived from various sources together with any interaction effects, b) environmental factors such as temperature and wind influencing pollutant diffusion and receptor sensitivity as well as c) data on the distribution and condition of receptors including communities and ecosystems.

been able to improve matters. There seems to be no way in which we can get the information necessary to implement the Pigouvian tax-subsidy approach to the control of externalities” (Baumol and Oates, 1971).

Several approaches have been investigated in order to provide policy recommendations on instrument choices when marginal damage cannot be estimated. Baumol and Oates (1971), for example, suggest imposing standards for an acceptable environment and a set of charges to achieve the desired level of pollution (see also e.g. Bimonte 1999 and Ng 2004).

In the present paper we deal with the non-existence of any applicable optimal fine function. Such a fine function could obviously not be properly implemented if the damage function cannot be estimated. We will, however, assume that the shape and specification of the damage function *is* fully known ex-ante. Note that even if we know exactly which factors contribute to the damage caused, how these factors interact and how large their relative effect is in general, it can still be virtually impossible to correctly predict the actual damage caused by any specific offense since accurate information on all contributing factors relevant to that offense would be needed. Our point is that, even under the strong assumption that the damage function is perfectly known, an applicable optimal fine function does not exist since the damage function is typically non-linear. We now argue that actual damage functions are typically complex, and thus almost never linear.³

The relationship between a firm’s emissions and its possible damage on nature and humans depends on involved processes. Economists often assume that nature is capable of handling or “cleaning up” small amounts of emissions, while ambient concentrations can be permanently and irreversibly affected if some discharge threshold is exceeded. The ambient concentrations may also vary with the location of the polluter and depend upon the recipient at risk. There could also be complex interactions between the concentrations of pollutants in one part of the environmental system, like the atmosphere, and other parts of the system, like oceans. The effects of emissions on changes in ambient concentrations are thus complex (Bolin 2003). In the case of greenhouse gas discharges, for example, ambient concentrations are not simply a linear function of discharges. The effect on atmospheric concentrations of additional emissions depends on the ability of e.g. biomass and oceans to absorb greenhouse gases; an ability that itself can depend on atmospheric concentrations (Wallace and Hobbs 2006, Ch. 10, IPCC 2001, 2007).

Further, the effect of ambient concentrations on nature, society and human well-being is clearly compound. While high concentrations of a pollutant in a desolated lake may have negligible impacts on human health, human health may be severely deteriorated by intermediate concentrations of air pollutants in a city. The *type* of pollution, as well as the interaction between different pollutants, may also affect human well-being. The health effect of pollution is thus clearly complex (e.g. Repetto 1987, IPCC 2007), and the same holds to a large extent with respect to costs to society in terms of the need to restructure or move capital (e.g. IPCC 2007). For example, the effect of higher global temperature due to climate change on damage is extremely involved. Some countries may expect some gains, while other countries may *vanish*. For small changes in temperature, the costs may not be very high, while costs may explode if a threshold is exceeded.

³ One possible exception we can think of is small, local, non-cumulative pollution caused by one single polluter, possibly such as noise or bad smell associated with the production processes.

Finally, the possibility of interactions between the complex economic and environmental systems can increase the complexity many-fold (Hanley and Spash 1993, p. 158). Interactions between human and environmental systems can be non-linear and chaotic, and, due to stochastic processes, sometimes even completely unpredictable (Bolin 2003). Returning to the greenhouse gas example, what environmental damages are observed would impact on what remedial measures are agreed upon by the global society. And the other way around, the way parts of the ecosystem are responding depends on when and how society implements remedies to handle global warming.

Serious attempts to estimate damage functions also conclude that damage is not linear in emissions. Examples of such studies are the estimation of wildlife-inflicted property damage by Yoder (2002), the tsunami damage estimations by Koshimura and Yanagisawa (2007) and the erosion damage functions estimated by Ananda et al. (2001).

In summary, both theoretical and empirical arguments underline that damage functions are complex, and hardly ever linear. While it might be possible to use a linear approximation of the damage function for a small range of emission levels, we will show next that the policy maker will a priori not be able to include the optimal fine function in rules or guidelines for all relevant emission levels.

3. Model

First we mention the assumptions made in the model. Next we look into the behavior of the firms and the regulator in order to derive the optimal fine function. We will see that when the damage function is linear we replicate the classical result of the existence of an optimal fine. When allowing for non-linear damage functions, however, this classical result no longer holds.

3.1 Model assumptions

We consider a static model where firms produce a homogenous good X which can be sold at a market price $p(X)$. Each firm i produces an amount x_i of the good and $\sum_i x_i = X$. We assume

that all firms have identical production costs: $C^x(x)$ with $\frac{dC^x}{dx} > 0$ and $\frac{d^2C^x}{dx^2} > 0$. However, as a byproduct of production, emissions are discharged into the environment: each unit of production leads to a discharge of e_o units of the pollutant.

Now, the regulator decides to impose an emission standard $\bar{e} < e_o$ per unit of production on the firms. This will make firms consider their abatement options so as to reduce their emissions and comply with the environmental regulation. The type of abatement costs we consider are more like operation costs and not investment costs. Firms have different abatement costs $C_i^a(a_i)$ per unit of production where a_i represents the reduction in emissions and $\frac{dC_i^a}{da} > 0$. So the firm's residual emissions equal $e_i = e_o - a_i$. These residual emissions

cause damage $D(E)$ to the environment where E is the total of emissions produced by all the firms⁴:

$$E = \sum_i x_i [e_o - a_i] = \sum_i x_i e_i .$$

In order to enforce the environmental regulation, firms are randomly inspected by the environmental inspection agency with probability π . Furthermore, when a firm is found to be in violation, a fine $F(a_i)$ is imposed. This fine depends on the level of abatement⁵ in firm i :

$$\begin{aligned} F(a_i) &> 0 && \text{if} && e_o - a_i > \bar{e} \\ F(a_i) &= 0 && \text{if} && e_o - a_i \leq \bar{e} \end{aligned}$$

with e_o representing the emission level that would have been optimal for the firm in the situation without emission limit.

3.2 Model

First we model the firms' decision with respect to production and abatement levels. Next we turn to the regulator's choice concerning the design of the optimal fine function.

Firm decision

Firms maximize profit Π_i by choosing production levels x_i and abatement levels a_i .

$$\max_{a_i, x_i} \Pi_i = \max_{a_i, x_i} [p(X)x_i - C^x(x_i) - C_i^a(a_i)x_i - \pi F(a_i)x_i]$$

We can now derive the first order condition with respect to x_i :

$$p(X) + \frac{dp(X)}{dx_i} x_i = \frac{dC^x(x_i)}{dx} + C_i^a(a_i) + \pi F(a_i) \quad (1)$$

So, optimal production is such that the marginal increase in sales equals the marginal expected costs associated with production.

Looking at the abatement decisions, firms will comply with the emission standard \bar{e} when the marginal expected penalty for non-compliance is larger than the abatement costs savings of marginally exceeding the standard; that is, when⁶

$$\pi \frac{d^+ F}{da}(e_o - \bar{e}) \geq -\frac{dC_i^a}{da}(e_o - \bar{e})$$

⁴ Overall damage is likely to be affected not only by the (aggregate) emission levels but also by other factors such as prevention efforts by victims, the state of the environment or weather influences. Since these additional factors are assumed to be exogenous to the firms' as well as the government's decision processes, we can express the damage function D as a function of aggregate emissions by firms alone without compromising the model.

⁵ This is equivalent to specifying the fine as a function of emissions since a firm's per unit emissions are $e_i = e_o - a_i$.

⁶ Here d^+ represents the right-hand side derivative of the fine function in point $e_o - \bar{e}$.

However, the optimal strategy is to exceed the standard if the marginal expected penalty is below the marginal abatement cost savings at the standard. In that case, we have an interior solution and the first order condition is

$$-\frac{dC_i^a}{da} x_i = \pi \frac{dF}{da}(a_i) x_i \quad (2)$$

This is the traditional condition that the firm will abate until the marginal expected penalty for non-compliance is equal to the marginal abatement costs savings of exceeding the standard.

Regulator

The regulator maximizes social welfare, SW , with respect to the fine function $F(a)$. We denote the consumer surplus associated with the good X as $CS(X)$ and assume that inspecting a firm costs m per site visit, and that imposing a fine on a detected violator entails no costs. Thus the regulatory objective function can be written as

$$\max_{F(a)} SW = \max_{F(a)} \left\{ \sum_i \Pi_i(x_i, a_i) + CS(X) - D(E) - \sum_i m\pi + \sum_i \pi F(a_i) x_i \right\}$$

The regulator wants to select the fine function in such a way that the firms choose the optimal production and abatement levels. Therefore, we first derive the socially optimal production and abatement levels. The relevant first order conditions are

$$\begin{aligned} \frac{d\Pi_i}{dx} + \frac{dCS}{dx} - \frac{dD}{dx} + \pi F(a_i) &= 0 \quad \text{and} \\ \frac{d\Pi_i}{da} - \frac{dD}{da} + \pi \frac{dF}{da}(a_i) x_i &= 0 \end{aligned}$$

These expressions can be rewritten as follows:

$$\begin{aligned} p(X) + \frac{dp(X)}{dx} x_i - \frac{dC^x(x_i)}{dx} - C_i^a(a_i) &= \frac{dD}{dx} - \frac{dCS}{dx} \\ \frac{dC_i^a}{da} x_i &= \frac{dD}{da} \end{aligned}$$

Next we compare these expressions with the first order conditions for the firms' production and abatement decisions expressed in equations (1) and (2). This allows us to formulate the necessary conditions to be fulfilled by the fine function in order to ensure optimal production and abatement levels. Namely

$$\pi F(a_i) = \frac{dD}{dx} - \frac{dCS}{dx} \quad (3)$$

$$\pi \frac{dF}{da}(a_i) x_i = \frac{dD}{da} \quad (4)$$

Hence the optimal expected fine per unit of production must equal the marginal damage of an additional unit of production corrected for the marginal effect on consumer surplus. Moreover, the expected marginal fine has to be equal to the marginal environmental harm of an additional unit of abatement for all produced goods.

In order to exist, the optimal fine function needs to satisfy both of these conditions. We note that they imply a specific relationship between the shape of the fine function and the shape of the damage function. Before we establish that the optimal fine function does not exist if the damage function is non-linear, we look at simpler damage functions.

Firstly, we examine the simple case where the damage function is a constant. This is obviously an unrealistic case since it would imply that an infinitesimally small amount of the pollutant discharged into the environment has exactly the same impact as an infinitely large discharge of that pollutant. Thus the first derivatives with respect to x and a are zero in this case. Consequently we obtain a corner solution: either the optimal fine is zero if the environmental damage is less than the benefit of production for consumers and producers, or the optimal fine is a large constant so as to ensure perfect compliance with the emission standard.

Secondly, we consider a linear damage function with respect to emissions:

$$D(E) = \gamma + \delta E = \gamma + \delta \sum_i x_i [e_o - a_i]$$

As we have already mentioned, a linear approximation of the damage function might be valid for small emission ranges but are unlikely to hold for larger ranges.

We now have:

$$\frac{dD}{dx} = \delta [e_o - a_i] > 0 \quad \text{and} \quad \frac{dD}{da} = -\delta x_i < 0$$

The optimal fine thus has to fulfill the following conditions:

$$\pi F(a_i) = \delta [e_o - a_i] - \frac{dCS}{dx} \quad \text{and} \quad \pi \frac{dF}{da}(a_i) x_i = -\delta x_i$$

that can equivalently be written as follows:

$$F(a_i) = \frac{\delta e_o - \frac{dCS}{dx}}{\pi} - \frac{\delta}{\pi} a_i \quad \text{and} \quad \frac{dF}{da}(a_i) = -\frac{\delta}{\pi}$$

These two conditions perfectly define the optimal fine function that is a linear function of abatement effort. Here we replicate the classical expression for the optimal marginal fine: i.e. the optimal marginal fine should equal the marginal damage divided by the probability of detection. Further, we see that the functional form of the fine function mimics the functional form of the damage function. Indeed, a constant damage function implies a constant optimal fine, while a linear damage function implies a linear optimal fine. However, for damage

functions that are non-linear, there exists no fine function that can fulfill the optimality conditions (3) and (4), as we state in the following proposition

Proposition: *The optimal fine as a function of the prosecuted firm's abatement effort does not exist if the environmental damage function is non-linear in total emissions.*

Proof: If we derive the damage function $D(E)$ with respect to x_i , we have (see expression (3)):

$$\pi F(a) = \frac{dD}{dx} - \frac{dCS}{dx} = \frac{dD}{dE} \frac{dE}{dx} - \frac{dCS}{dx} = \frac{dD}{dE} [e_o - a_i] - \frac{dCS}{dx} \quad \text{and} \quad \text{thus}$$

$$\pi \frac{dF}{da} = -\frac{dD}{dE} - [e_o - a_i] \frac{d^2 D}{dE^2} x_i. \quad \text{From expression (4), we have}$$

$$\pi \frac{dF}{da}(a_i) x_i = \frac{dD}{da} = \frac{dD}{dE} \frac{dE}{da} = -\frac{dD}{dE} x_i. \quad \text{Hence we find that in order to fulfill the optimality}$$

conditions for the fine function, it must hold that $-\frac{dD}{dE} - [e_o - a_i] \frac{d^2 D}{dE^2} x_i = -\frac{dD}{dE}$. This implies

that the second derivative of the damage function must be zero: $\frac{d^2 D}{dE^2} = 0$.

QED.

In other words, it is only possible for constant or linear damage functions to optimize the fine as a function of the prosecuted firm's abatement efforts. A linear damage function implies that, among other things, there are no threshold effects, no interactions between the emissions of the different firms in the regulated industry and no cumulative pollution problems. Therefore it is possible to simply add the emissions, multiply the sum by a constant and calculate the total damage caused.

When the damage function is no longer linear, it is impossible to assess the impact of one firm on total damage by only referring to that particular firm's emissions. For instance, in the case of threshold effects, it will be necessary to assess previous concentrations in the environment before being able to assess the actual damage caused. Also, in the case of interactions with emissions of other firms, it will be necessary to refer to the emissions of these other firms in order to determine the actual damage that occurred. So it is not possible to design an optimal fine as a function of the prosecuted firm's abatement efforts. Such a fine function would not take the interdependencies between firms, economy and environment into account and would therefore be non-optimal. For this reason, it will not be possible to induce optimal behavior by firms since the fine function is not able to replicate the marginal environmental damage function.

4. Other policy instruments

In this section we show that the result also holds for other environmental policy instruments. The formal derivations of firm and regulator behavior when confronted with an emission tax, an emission cap or a cap and trade system respectively can be found in the appendix.

Firstly we consider an emission tax. When confronted with an emission tax, firms can decide to underreport and report fewer emissions than their true emission level. However, firms face the possibility of tax audits and when tax evasion is detected during the audit, it is sanctioned. Consistent with previous models (e.g. Rousseau and Proost 2005) we find that the amount of emissions reported by the firm is such that the tax saved by not reporting an additional unit of emissions is equal to the increase in the marginal expected fine. From the regulator's objective function we find that the expected marginal fine for tax evasion has to be equal to the marginal environmental damage of an additional unit of abatement for all produced goods, corrected for the part of the tax the firm already paid. After all, the tax already paid by the firm partly internalizes the external damage caused by production and therefore the sanction should only be based on the remaining damage caused. This implies that the optimal fine function crucially depends on the shape of the environmental damage function, and hence our non-existence result also holds for emission taxes.

Next we assume that the government imposes an emission cap on each firm. So total emissions of each firm should not exceed a certain limit, otherwise they run the risk of getting fined. The regulator wants to motivate the firms to choose optimal production and abatement levels. Hence, the optimal expected fine must equal the marginal damage of an additional unit of emissions, at least as long as the marginal consumer surplus is zero. Again, this implies that the optimal fine function crucially depends on the shape of the environmental damage function, and hence our non-existence result also holds for emission caps.

Finally, we look at a cap and trade system⁷. The regulator imposes an emission cap on the whole industry, permits are assumed to be grandfathered⁸ (i.e. initial distribution of permits is free) and permits can be traded among firms. We assume that individual firms do not have market power in the permit market and are thus price takers. In a cap and trade system, firms can decide to hold fewer permits than needed to cover their actual emissions. However, firms are audited and face fines when their actual emissions are not fully covered by their permit holdings. In order to decide how many permits to hold, the firm will thus weigh the cost of buying an additional permit (or of not being able to sell one more permit) with the marginal savings in the expected fine. Also, the firm's optimal abatement level is chosen such that the marginal abatement costs equal the savings of not having to buy an additional permit (or having an additional permit to sell on the market) plus the savings in the expected marginal fine. From the regulator's objective function we find that optimal expected fine of holding an insufficient amount of permits must equal the marginal damage of an additional unit of production corrected for the marginal effect on consumer surplus. Moreover, the expected marginal fine has to be equal to the marginal environmental harm of an additional unit of abatement for all produced goods corrected for the part of the permits needed to cover actual emission the firm already obtained. After all, the permits already obtained by the firm partly internalizes the external damage caused by production and therefore the sanction should only be based on the remaining damage caused. This implies that the optimal fine function crucially depends on the shape of the environmental damage function, and hence our non-existence result also holds for cap and trade systems. Even though the fine function influences the permit price in the market through the firms' abatement and permit holding decisions, the

⁷ Other studies on the working of tradable permit systems in the presence of non-compliance include Montero (2002) and Stranlund (2007).

⁸ Assuming that permits would be auctioned would not substantially change the results. Firms' profits would be lower since they would not receive rents from the initial allocation of permits

permit market will not be able to correct fully for the presence of non-linear damage function, if the fine function cannot be optimally designed by the regulator.

5. Discussion

Our result crucially depends on the shape of the environmental damage functions and as we argued in section 2, environmental damages are typically related to firms' emission levels in a non-linear way. This non-linearity can be caused by interactions with environmental characteristics such as other sources emitting the same pollutant, emissions of other pollutants or the functioning of input and output markets. Interactions with other pollutants, cumulative pollution problems depending on the current state of environmental quality in receptor points and prevention efforts undertaken by victims are all important issues affected by the results of our model. In this section we first discuss further why a non-linear fine function is not implementable. Then we try to illuminate how deterrence errors caused by imperfect information could be enhanced by not accounting for non-linearities of damage functions. Finally, we discuss enforcement in a second-best setting, when the optimal fine approach is not feasible.

5.1. When *may* an optimal fine function exist?

Since damage functions are typically very complex, our theoretical model implies that an applicable optimal fine function would not exist. In our model, the basic reason for this non-existence result is that non-linearity of the damage function implies that the damage caused by emissions of one firm depends on additional firm-external factors such as emissions of other firms or ambient concentrations. Since an optimal fine function should mimic the curvature of the damage function, an optimal fine function may, however, exist in our model if the fine of one firm was allowed to depend on firm-external factors like emissions of all other firms or ambient concentrations. While it might be possible to boldly account for one or two firm-external factors, like the vulnerability of a firm's receptor, there are a number of reasons why it is in practice the fine function cannot depend on all relevant firm-external factors.⁹

Firstly, the information required to design a fine function that accounts for e.g. emissions of other firms would be tremendous. The regulator would not only need to know the emissions of the prosecuted firm, but also the level and distribution of the other firms' emissions at the time when the firm emitted excessively, and for cumulative pollution problems, also past emission levels. In fact, emissions from the firm would also interact with the ecosystem, as well as result in behavioral effects on the rest of the economy that must also in principle be taken into account. As a case in point, the fine function should depend on the flow and water level of the river to correctly reflect the harm caused by a wastewater discharge into it. This would include the type and timing of pollutants discharged, currently and previously, and how the given firm's emissions interact with discharges now and in the future. The fine function defined ex-ante by the regulator should also incorporate the illegal competitive advantage incurred by the firm not paying its compliance costs. Such a competitive advantage may lead to the overprovision of polluting goods in other markets, and the effect of this (and interactions) should be reflected by the fine function. Computing this advantage is, however,

⁹ In practice many fine functions depend on the violator's own behavior. For example, a violator's speeding fine depends on the extent of violation of the speed limit, and not on the extent that other drivers also speed excessively. Also, the EPA's Clean Air Act Stationary Source Civil Penalty Policy (1991) describes civil fines for violation of air pollution standards as '\$5000 for each 30% or fraction of 30% increment above the standard' – i.e. independent of firm-external factors.

extremely challenging, as illustrated by the fact that the US EPA considers ‘a model to handle such calculations (to be) infeasible’ (EPA 2005).

Secondly, including factors in the fine function that depend on decisions made by third parties introduces uncertainty. The firm does not know *ex-ante* what others will decide and thus what damage it would cause if it violates the standard. Investment decisions (abatement) by firms become more difficult if the potential savings in terms of avoided fines are uncertain. Moreover, if the firm is responsible for the damage caused by its emissions no matter what the behavior of other parties, then these other parties will have no incentive to take costly prevention efforts. The firm then has to face the complete risk caused by its activities (cf. strict liability). This might lead to overinvestment by firms in emission reducing activities (especially if firms/managers are risk averse) and underinvestment by parties that can reduce the damage caused by a violation.

Finally, letting the fine depend on the emissions of other firms may be hard to reconcile with fundamental legal principles of non-discrimination. If two firms make the same discharge of a particular pollutant, but one does it one second before the other, such a fine function would imply that the second firm should be punished more severely than the first – and possibly a lot more severely. The authorities would have difficulties communicating that such different fines for similar behavior is just.¹⁰

Summing up, the application of optimal fine functions that correctly reflect the complexity of actual damage functions seems completely impracticable.

5.2. Non-linearities and imperfect information

It has been recognized for decades that limited knowledge of damage functions makes it hard to implement optimal fines (Baumol and Oates 1971). In our theoretical model we have made the assumption that damage functions are perfectly known, but that optimal fines are still not applicable if damage functions are non-linear. One may ask whether the first problem (i.e. imperfect information on damage functions) is so insurmountable that neglecting the second (i.e. non-linearity) would not really matter for real-life policy. This seems a question that is very hard to answer satisfactorily. Here we argue that the impact on applicable fines of imperfect information and non-linearities are interconnected. The deterrence errors caused by imperfect information could be amplified when non-linear damage functions are erroneously assumed to be linear.

As a simple illustration of this point consider a specific form of non-linear damage function where a firm causes excessive noise. The damage caused by the noise can, for example, depend on the type of windows installed in surrounding houses (like single or double panes of glass). Thus the optimal fine function would depend not only on the level of noise caused by the violator, but also on the actions taken by external parties, i.e. the inhabitants of the surrounding houses.

Consider the situation when this damage function is known with certainty by the regulator. The optimal fine function would then be firm-specific and depend on the firm’s noise levels and the *ex-ante* proportion of surrounding house-owners with sound-proof glass. However, if the fine function is not allowed to depend on the firm-specific proportion of houses with

¹⁰ Admittedly, it might be *just* under the extremely unrealistic assumption that all firms know everything about the emissions, as well as the effect of these emissions on harm, of all other firms.

sound-proof glass, the optimal fine would not be applicable. Assume, for example, that instead of the firm-specific proportion of houses with sound-proof glass, it is only feasible to use a regional proportion. This implies that on average the expected fine equals the optimal fine, but, some firms will be overdeterred and others will be underdeterred since the real proportion of sound-proof investments in the violator's neighborhood is likely to differ from the regional average.

Consider, next, the situation when this damage function is unknown. Then the regulator will have to rely on some estimate of the damage function, which can be estimated based on past observations of various noise levels and damages. In regression models where damages are assumed to be linear, actions by house-owners are ignored and the damage function can be estimated by a univariate linear regression. However, we could reduce the error associated with imperfect information by explicitly taking non-linearities into account, i.e. rather than estimating a regression based on firms' emissions only, the estimated damage function could also explicitly take non-linearities into account by including factors such as 'sound-proofing' as explanatory variable. Such a regression model with interactions would then reduce the error caused by imperfect information and reduce over- and underdeterrence of potential violators. Thus, even with imperfect information, if non-linearities are ignored, deterrence errors will be larger compared to cases where damages would be a linear function in terms of a firm's own emissions.

5.3. What do we do without optimal fines?

Following our theoretical model and confronting it with actual damage functions, fine functions that are optimal and applicable would (almost) never exist. This result reduces the attractiveness of fines and, to the extent that policymakers somehow experience this drawback, it might help explain why enforcement agencies and courts have frequently applied other enforcement instruments.

If the fine function that is imposed is not optimal, deterrence will also not be optimal. Some firms will pollute excessively, while others might over-invest in abatement. In the presence of such uncertainty, other enforcement instruments may become relatively more attractive even though they are not likely to be socially optimal. Uncertainty with respect to the potential harm that can be caused by excessive emissions will make enforcement actions that directly affect the quantity of emissions, such as plant closures and regularization orders, more attractive compared to instruments that only attach a price to non-compliance, such as fines. After all, when fines cannot be optimally designed, the most ambitious goal a regulator might have is to avoid really harmful situations. This is in keeping with the precautionary principle. Also it has a similar flavor as Weitzman's finding that quantity instruments might be preferable if environmental benefits are uncertain (Weitzman 1974).

A related option is to use inspections in order to provide improved compliance incentives to firms. It is after all possible to differentiate the stringency of control according to firms' characteristics - like general pollution level of the firm's industry, the environmental vulnerability of firm location, or the firm's past non-compliance record. Indeed this type of targeting is often used in reality (see e.g. Gray and Deily 1996, Stafford 2002, Nyborg and Telle 2006, Rousseau 2007, Telle 2009). Due to informational limitations and the need for some consistency in day-to-day monitoring policies, using such differentiated inspection strategies will not provide a perfect solution for the non-existence of optimal fines. However, differentiated inspections may help to limit the social loss of not being able to impose optimal

fine functions and may therefore be a vital part of environmental monitoring and enforcement policies.

Under the unrealistic assumption that the regulator is capable of estimating the non-linear damage function, a second-best fine function¹¹ that does not depend on the emissions of other firms may be implemented as follows.¹² At the beginning of each period, the agency announces a linear fine function that is a linear approximation to the non-linear damage function around the relevant area of total emissions. After some periods, the agency should be able to set the linear fine function so that it is close to the marginal harm. The disadvantage of this approach is that the agency may have to change the fine function frequently, which can imply huge costs of adaptation on firms, and which can be politically difficult (Cropper and Oates 1992). Moreover, policy uncertainty affects firms long-term investment decisions since their expected returns are uncertain. Also, if a linear approximation is used, the probability of conflict, discussions and appeal is increasing since the true damage is likely to differ from the estimated damage ex-ante.

Agencies or judges typically have substantial discretion with respect to how a detected violator is punished; and in the case of fines, this discretion also includes the *size* of the fine. Concerns that individuals committing similar crimes are not also punished similarly have led legal scholars and policymakers to argue for and to implement less judicial discretion (cf. ongoing discussion regarding *Kimbrough v. U.S.*, No. 06-6330). The extent of judicial discretion has also received some attention in the law and economics literature (Miceli 2008, Shavell 2007, Reinganum 2000). Though we argue that specifying the optimal fine function before emissions occur seems impossible, the damage caused by a given discharge might be somewhat easier to estimate ex-post. In this sense, our model indicates an argument in favor of judicial discretion.

6. Conclusion

We have shown that an applicable fine function, which implements the socially optimal level of production and emissions for environmental regulation would almost never exist. The reason is that damage functions are almost never linear, and that the fine imposed on one firm cannot depend on all relevant factors including the emissions of all other firms and ambient concentrations. As shown, the issue is not only whether damages depend on individual or aggregate emissions, but also whether the damage function is linear or non-linear in emissions. So, even if individual emissions of different sources can be summed in an additive way to an aggregate emission level, the problem is not solved (and the optimal fine function does not exist) if damages are a non-linear function of these aggregate emissions. Moreover, as we point out, emissions of different sources are often not additive and thus the damage function is even more likely to be a non-linear function of individual emissions.

Previous literature on optimal fines does not recognize the non-existence of the applicable optimal fine since this literature typically uses discrete models where the (marginal) damage

¹¹ Another approach to designing a second-best fine function in the case where the non-linearity arises from interactions between firms' emissions of a particular pollutant leading to non-separable damage, can be inspired by the literature on cost-sharing (Moulin and Shenker 1992). However, note that this approach addresses only one of the causes of non-linearity in the damage functions.

¹² This is in line with the argument made by Kaplow and Shavell (2002) in the case of non-linear tax schemes. This argument is also similar in spirit to the approach of Roberts and Spence (1976) where a step-wise linear fine function is applied to approximate a non-linear damage function.

caused by the crime is assumed constant. Our arguments reduce the attractiveness of fines, and may thus help explain why other enforcement instruments are frequently applied by enforcement agencies.

Appendix

Emission tax

We assume now that the regulator imposes an environmental tax t per unit of emissions produced by the firm.

Firm decision

Firms maximize profit Π_i by choosing production levels x_i , abatement levels a_i and the amount of reported emissions r_i :

$$\max_{a_i, x_i, r_i} \Pi_i = \max_{a_i, x_i, r_i} \left[p(X)x_i - C^x(x_i) - C^a(a_i)x_i - tr_i x_i - \pi F(e_i - r_i)x_i \right]$$

For ease of derivation we express the abatement costs, the reported emissions r_i , the actual emissions e_i and the fine function per unit of production. This does not change the results.

We can now derive the first order condition with respect to x_i :

$$p(X) + \frac{dp(X)}{dx} x_i - \frac{dC^x(x_i)}{dx} - C^a(a_i) - tr_i = \pi F(a_i, r_i)$$

Next we derive the optimal amount of emissions the firm will report. This is determined by the condition $t = \pi \frac{dF(a_i, r_i)}{dr_i}$. The amount of emissions the firm should report is such that the

tax saved by not reporting is equal to the increase in the marginal expected fine. Using this expression, we derive the first order condition with respect to a_i :

$$\frac{dC^a}{da} x_i = t \frac{dr_i}{da} x_i - \pi \frac{dF}{da}(a_i, r_i) x_i$$

Optimal abatement is chosen such that the marginal abatement costs equal the savings in the marginal tax paid, i.e. the net tax rate, and in the expected marginal fine.

Regulator

The regulatory objective function can be written as

$$\max_{F(a,r)} SW = \max_{F(a,r)} \left\{ \sum_i \Pi_i(x_i, a_i) + CS(X) - D(E) - \sum_i m\pi + \sum_i \left[\pi F(a_i, r_i) x_i + tr_i x_i \right] \right\}$$

We derive the socially optimal production, reporting and abatement levels. The relevant first order conditions are

$$\frac{d\Pi_i}{dx} + \frac{dCS}{dx} - \frac{dD}{dx} + \pi F(a_i, r_i) + tr_i = 0$$

$$t - \pi \frac{dF}{dr}(a_i, r_i) = 0$$

$$\frac{d\Pi_i}{da} - \frac{dD}{da} + \pi \frac{dF}{da}(a_i, r_i) x_i - t \frac{dr_i}{da} x_i = 0$$

We can now formulate the necessary conditions to be fulfilled by the fine function in order to ensure optimal production, reporting and abatement levels. Namely

$$\pi F(a_i, r_i) = \frac{dD}{dx} - \frac{dCS}{dx} - tr_i,$$

$$\pi \frac{dF}{dr}(a_i, r_i) = t$$

$$\pi \frac{dF}{da}(a_i, r_i) x_i = \frac{dD}{da} + t \frac{dr_i}{da} x_i$$

Hence the optimal expected fine per unit of production must equal the marginal damage of an additional unit of production corrected for the marginal effect on consumer surplus. Moreover, the expected marginal fine has to be equal to the marginal environmental harm of an additional unit of abatement for all produced goods corrected for the part of the tax the firm already paid.

Emission cap

Assume that the government imposes an emission cap on each firm. So total emissions of each firm should not exceed a certain limit, otherwise they run the risk of getting fined.

Firm decision

Firms maximize profit Π_i by choosing production levels x_i and abatement levels a_i .

$$\max_{a_i, x_i} \Pi_i = \max_{a_i, x_i} [p(X)x_i - C^x(x_i) - C_i^a(a_i)x_i - \pi F(E_i)]$$

with total emissions of firm i equal to $E_i = x_i e_i = x_i [e_o - a_i]$.

We can now derive the first order condition with respect to x_i :

$$p(X) + \frac{dp(X)}{dx_i} x_i - \frac{dC^x(x_i)}{dx_i} - C_i^a(a_i) = \pi \frac{dF(E_i)}{dE_i} [e_o - a_i]$$

The first order condition with respect to a_i is

$$-\frac{dC_i^a}{da} x_i = -\pi \frac{dF(E_i)}{dE_i} x_i$$

Regulator

The regulatory objective function can be written as

$$\max_{F(a)} SW = \max_{F(a)} \left\{ \sum_i \Pi_i(x_i, a_i) + CS(X) - D(E) - \sum_i m\pi + \sum_i \pi F(E_i) \right\}$$

The regulator wants to select the fine function in such a way that the firms choose the optimal production and abatement levels. This allows us to formulate the necessary conditions to be fulfilled by the fine function in order to ensure optimal production and abatement levels. Namely

$$\pi \frac{dF(E_i)}{dE_i} [e_o - a_i] = \frac{dD}{dE} [e_o - a_i] - \frac{dCS}{dx} \quad \text{and} \quad -\pi \frac{dF(E_i)}{dE_i} x_i = -\frac{dD}{dE} x_i$$

Hence if $\frac{dCS}{dx} = 0$, the optimal expected fine must equal the marginal damage of an additional unit of emissions. Again this will only hold if there are no interactions in the environmental damage function.

Tradable permits – Cap and trade

We assume now that the regulator imposes an emission cap on the whole industry, permits are grandfathered (i.e. initial distribution of permits l_o is free) and can be traded among firms. The long term permit price in equilibrium is represented by p_p . We assume that individual firms do not have market power in the permit market and are thus price takers.

Firm decision

Firms maximize profit Π_i by choosing production levels x_i , abatement levels a_i and the amount of permits they hold l_i :

$$\max_{a_i, x_i, l_i} \Pi_i = \max_{a_i, x_i, l_i} \left[p(X) x_i - C^x(x_i) - C^a(a_i) x_i - p_p [l_i - l_o] x_i - \pi F(e_i - l_i) x_i \right]$$

For ease of derivation we express the abatement costs, the permits holdings l_i , the actual emissions e_i and the fine function per unit of production. This does not change the results.

We can now derive the first order condition with respect to x_i :

$$p(X) + \frac{dp(X)}{dx} x_i - \frac{dC^x(x_i)}{dx} - C^a(a_i) - p_p [l_i - l_o] = \pi F(a_i, l_i)$$

Next we derive the optimal amount of permits held by the firm. This is determined by the condition $p_p = \pi \frac{dF(a_i, l_i)}{dl_i}$. The amount of permits the firm should hold is such that the

amount saved by not buying an additional permit is equal to the increase in the marginal expected fine. Using this expression, we derive the first order condition with respect to a_i :

$$\frac{dC^a}{da} x_i = p_p \frac{dl_i}{da} x_i - \pi \frac{dF}{da}(a_i, l_i) x_i$$

Optimal abatement is chosen such that the marginal abatement costs equal the savings of not having to buy an additional permit (or having an additional permit to sell on the market) plus the savings in the expected marginal fine.

Regulator

The regulatory objective function can be written as

$$\max_{F(a,l)} SW = \max_{F(a,l)} \left\{ \sum_i \Pi_i(x_i, a_i) + CS(X) - D(E) - \sum_i m\pi + \sum_i [\pi F(a_i, l_i) x_i + p_p l_i x_i] \right\}$$

We derive the socially optimal production, permit holding and abatement levels. The relevant first order conditions are

$$\frac{d\Pi_i}{dx} + \frac{dCS}{dx} - \frac{dD}{dx} + \pi F(a_i, l_i) + p_p [l_i - l_o] = 0$$

$$p_p - \pi \frac{dF}{dl}(a_i, l_i) = 0$$

$$\frac{d\Pi_i}{da} - \frac{dD}{da} + \pi \frac{dF}{da}(a_i, l_i) x_i - p_p \frac{dl_i}{da} x_i = 0$$

We can now formulate the necessary conditions to be fulfilled by the fine function in order to ensure optimal production, permit holding and abatement levels. Namely

$$\pi F(a_i, l_i) = \frac{dD}{dx} - \frac{dCS}{dx} - p_p [l_i - l_o],$$

$$\pi \frac{dF}{dl}(a_i, l_i) = p_p$$

$$\pi \frac{dF}{da}(a_i, l_i) x_i = \frac{dD}{da} + p_p \frac{dl_i}{da} x_i$$

Hence the optimal expected fine per unit of production must equal the marginal damage of an additional unit of production corrected for the marginal effect on consumer surplus. Moreover, the expected marginal fine has to be equal to the marginal environmental harm of an additional unit of abatement for all produced goods corrected for the part of the permits needed to cover actual emission the firm already obtained.

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